Evolution of $\nu = 1$ Bilayer Quantum Hall Ferromagnet

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The natures of the ground state in a $\nu_{\rm T} = 1$ bilayer quantum Hall system at a variety of layer spacing are investigated. At small layer separations the system exhibits spontaneous interlayer phase coherence. It is claimed that the Halperin's (1,1,1) state is not relevant in the incompressible regime near the incompressible to compressible transition point in which the Josephson-like effect was observed. The two-particle correlation function shows the deflated correlation hole at this regime. An effective model that can give a good approximation to the ground state is proposed. A connection to the modified composite fermion theory is discussed.

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The fractional quantum Hall effect which appears in a two-dimensional electron system in a strong magnetic field exhibits rich strong correlation phenomena. [1] For a decade, a large number of studies have been made on bilayer quantum Hall systems which consist of a pair of two-dimensional electron gases separated by a distance so small as to be comparable to the typical spacing between electrons in the same layer. [2] The characteristic parameters of this system are the filling factor $\nu_{\rm T} = 2\pi l^2 n$, the distance between the layers d, and the interlayer tunneling rate $\Delta_{\rm SAS}$. Here n is the total electron density of the system and l is the magnetic length. This system can be mapped to an equivalent spin-1/2 system by assigning $\uparrow(\downarrow)$ pseudo spins to electrons in the upper (lower) layer, where the actual electron spins are assumed to be polarized. In this system, various ground states and spectacular excitations are realized depending on the parameters. The Coulomb interaction term consists of direct and exchange terms. Because the latter term tends to align the pseudo spins of the electrons, the ground state has a ferromagnetic long range order at $\nu_{\rm T} = 1/q$ (q odd) when $d \simeq 0$. At finite layer separation, the direct part of the electron-electron interaction produces a local capacitive charging energy that is minimized when the two layers have equal electron density and cause the easy-plane(XY) type symmetry in the pseudospin space. The ferromagnetic order corresponds to the spontaneous interlayer phase coherence.

The simplest Abelian quantum Hall states for the bilayer systems at $\nu_{\rm T} = 2/(m+n)$ are described by the two-component generalization of the Laughlin wave function first introduced by Halperin. [3]

$$\Psi_{m,m,n} = \prod_{i<j}(z_i^\uparrow - z_j^\uparrow)^m \prod_{i<j}(z_i^\downarrow - z_j^\downarrow)^m \prod_{i,j}(z_i^\uparrow - z_j^\downarrow)^n {\rm e}^{-\frac{1}{4}\sum(|z_i^\uparrow|^2+|z_i^\downarrow|^2)}. \tag{1}$$

Based on the numerical diagonalization, Yoshioka *et al.* investigated that these variational wave functions give a good approximation to the ground state at a certain d and $\nu_{\rm T}$. [4] In the case of small layer separation at $\nu_{\rm T} = 1$, the ground state is $\Psi_{1,1,1}$ state with the ferromagnetic order which can also be regarded as an excitonic state.

On the other hand, for an infinite separation, the bilayer systems at $\nu_{\rm T} = 1$ reduces to two compressible uncorrelated $\nu = 1/2$ systems. This compressible state was discussed by Halperin *et al.* [5] as a Fermi-liquid state of the composite fermions. [6] The approximated wave function in this limit may be written as a simple product:

$$\Psi_{\rm IFL} = \Psi_{\rm FL}(\{z^\uparrow\})\Psi_{\rm FL}(\{z^\downarrow\}). \tag{2}$$

This state is called the independent Fermi-liquid state. Here $\Psi_{\rm FL}(\{z\}) = P_{\rm LLL}{\rm det}M\prod_{i<j}(z_i - z_j)^2\ {\rm e}^{-\frac{1}{4}\sum_i|z_i|^2}$ is the wave function of a Fermi-liquid-like compressible state at $\nu = 1/2$, [7] and $P_{\rm LLL}$ is the projection operator to the lowest-Landau-level, ${\rm det}M$ is a Slater determinant with the plane-wave matrix elements $M_{ij} = {\rm e}^{{\rm i}k_i r_j}$. Varying the layer separation, the quantum Hall to no quantum Hall transition occurs. [8,9] Based on the Chern-Simons effective field theory, several predictions have been proposed theoretically. [10–12] In a recent experimental study, Spielman *et al.* [13] observed resonant interlayer tunneling at zero-bias in this system. They also have recently observed the quantized Hall drag. [14] Because the peak of the differential conductance and the quantized Hall drag are observed near the transition point, the properties of the ground state in this regime should be clarified. Several scenarios have been proposed for the evolution of the ground state as the layer separation increases from zero to infinity. The existence of the charge-density-wave state with the interlayer coherence near transition point is considered in the framework

of the Hartree-Fock approximation [15,16] and the effective field theory. [17] Bonesteel *et al.* postulate the pairing of the composite fermions of isolated $\nu = 1/2$ layers and obtained a charge gap inversely proportional to d^2 which cause the quantum Hall effect. [18] Kim *et al.* discussed the paired states in detail and proposed several candidates of the ground state wave function. [19] In numerical studies, Schliemann *et al.* suggested a single-phase transition in the pseudospin degrees of freedom, [20] and Nakajima found that the spectral function changes qualitatively around transition point. [21] The estimated critical value agrees with the experiment by Spielman *et al.*. However we do not have a clear understanding of the nature of the ground state around the transition point. In this letter, using the numerical diagonalization on a torus geometry, we study the details of the evolution of the bilayer system between small and large d/l regime in the absence of interlayer tunneling.

The kinetic part of the Hamiltonian quenches into the lowest Landau level, so we consider the interaction Hamiltonian in a projected Hilbert state into the lowest Landau level:

$$H = \frac{1}{\Omega} \sum_{\mathbf{q}\sigma\sigma'} \frac{1}{2}\, V_{\sigma\sigma'}(\mathbf{q}) \rho_\sigma(-\mathbf{q}) \rho_{\sigma'}(\mathbf{q}), \tag{3}$$

where $\sigma,\ \sigma'$ are pseudospin indices, $\rho_\sigma(\mathbf{q})$ is the density operator in the layer σ, and $V_{\sigma\sigma'}(\mathbf{q})$ is the Fourier component of $V_{\sigma\sigma'}(r) = e^2/\epsilon\sqrt{r^2 + d^2\delta_{\sigma,-\sigma'}}$. When the layer separation d vanishes, the Hamiltonian has the pseudospin SU(2) rotational symmetry that means $[H, \mathbf{S}] = 0$. Here $\mathbf{S}$ is the total pseudospin operator,

$$\mathbf{S} = \sum_{m=1}^{N} c^\dagger_{m\sigma} \frac{\boldsymbol{\sigma}_{\sigma\sigma'}}{2} c_{m\sigma'}, \tag{4}$$

where m means a single particle electron state and N is the number of electron. Then the ground state $|\Psi\rangle$ can be also the eigen state of the $\mathbf{S}^2$ and S_z;

$$\mathbf{S}^2|\Psi\rangle = \ S(S+1)\ |\Psi\rangle \tag{5}$$

$$S_z|\Psi\rangle = \left(\frac{N_\uparrow - N_\downarrow}{2}\right)|\Psi\rangle. \tag{6}$$

For ferromagnetic state $S = N/2$.

For the finite value of the layer separation d/l, the direct part with zero wave vector contributes as a static charging energy $(2d/N)S_z^2$ [8] which makes $S_z = 0$. Because the symmetry of the Hamiltonian is reduced from SU(2) to U(1), the ground state is no longer the eigen state of $\mathbf{S}^2$ although it still is an eigen state of S_z. We define the length of the total pseudospin by the expectation value $\langle\Psi|\mathbf{S}^2|\Psi\rangle = S(S+1)$. In Fig.2 the value of S/N as a function of d/l is plotted. The deviation from the ferromagnetic state with increase of d/l can be seen.

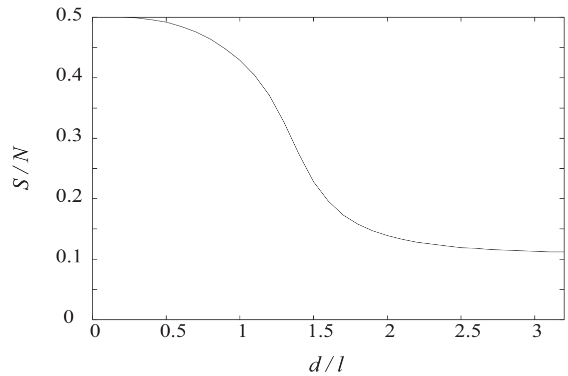

FIG. 1. The value of S/N, where S is the length of the total pseudospin, is plotted as a function of the layer spacing. The number of electron is $N = 12$.

To investigate the nature of the ground state we calculate the two-particle correlation function $g_{\sigma\sigma'}(\mathbf{r})$ which is defined as

$$g_{\sigma\sigma'}(\mathbf{r}) = \frac{1}{N(N-1)} \sum_{\mathbf{q}} \mathrm{e}^{\mathrm{i}\mathbf{q}\cdot\mathbf{r}}\ S_{\sigma\sigma'}(\mathbf{q}) \tag{7}$$

where

$$S_{\sigma\sigma'}(\mathbf{q}) = \langle\Psi|\rho_\sigma(-\mathbf{q})\rho_{\sigma'}(\mathbf{q})|\Psi\rangle \tag{8}$$

is the structure factor. Fig.2(a) shows the $g_{\uparrow\downarrow}(\mathbf{r})$ (solid line) and $g_{\uparrow\uparrow}(\mathbf{r})$ (dashed line) in the system with $d/l = 0.3$. Around the origin, both $g_{\uparrow\downarrow}(\mathbf{r})$ and $g_{\uparrow\uparrow}(\mathbf{r})$ behave as r^2. $g_{\uparrow\downarrow}(0) = 0$ shows a clear correlation hole. Such a behavior is the characteristic of the $\Psi_{1,1,1}$ state. Actually overlap between the $\Psi_{1,1,1}$ and the wave function of the exact ground state is almost unity in this regime. [4]

Next we consider the case $d/l = 0.9$. This regime may correspond to that in which the Josephson-like effect was observed in Spielman *et al*'s experiment. Actually Fig.1 shows that the ferromagnetic order or the interlayer phase coherence survives at this point. The system should be in a quantum Hall regime according to several experimental data [9,13] and theoretical analysis [8,12], but $\Psi_{1,1,1}$ is not relevant at this point. [4] The small but finite value of $g_{\uparrow\downarrow}(0)$ indicates that the correlation hole deflates gradually. The decline of the correlation hole may lead to non-singular behavior of the zero-bias conductivity. The $g_{\uparrow\downarrow}(\mathbf{r})$ in Fig.2(b) shows not only short range interlayer correlation but the fact that the electrons in the 'down' layer is attracted to the electron in the 'up' layer at the origin.

To understand this state we consider an effective model state which is defined as a ground state of the following pseudo potential interaction; [1] $V_l^{\uparrow\uparrow} = 0, V_l^{\uparrow\downarrow} = -\delta_{l,1}$.

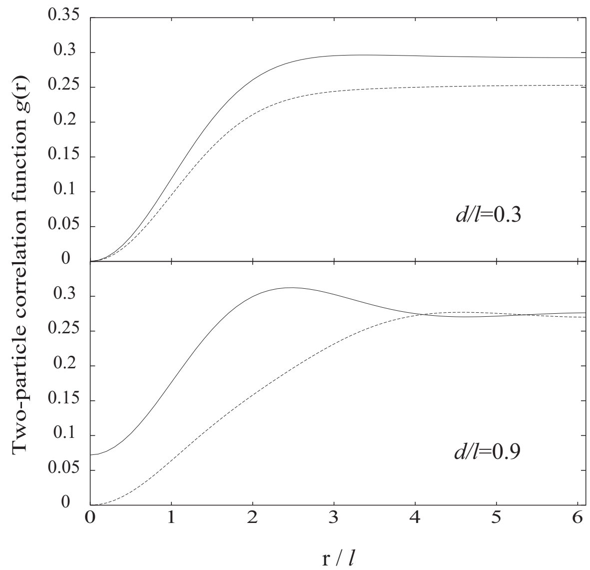

FIG. 2. The two-particle correlation function $g_{\uparrow\downarrow}$ (solid line), $g_{\uparrow\uparrow}$ (dashed line) at (a) $d/l = 0.3$, and (b) $d/l = 0.9$ are plotted. The total number of electron is $N = 12$.

The negative sign in $V_l^{\uparrow\downarrow}$ means interlayer attraction in the $l = 1$ channel of the angular momentum. This state has the imperfect correlation holes. On the other hand, $\Psi_{1,1,1}$ is defined as the ground state of the pseudo potential interaction $V_l^{\uparrow\uparrow} = V_l^{\uparrow\downarrow} = \delta_{l,0}$. In Fig.3, overlap between the exact ground state and these model states are shown. We find our effective model becomes relevant at $d/l = 0.85$. The maximum value of overlap is 0.996.

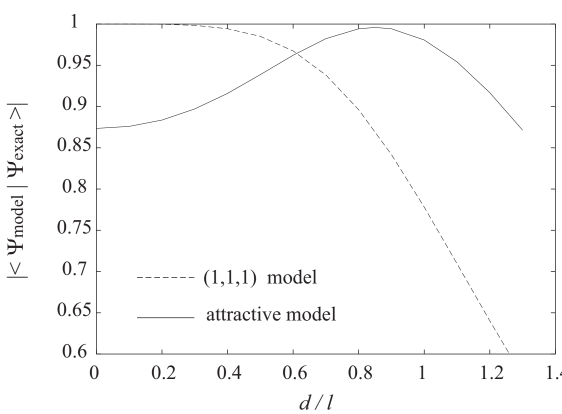

FIG. 3. Overlap between the exact ground state and the model states as functions of d/l. The number of electron is $N = 12$.

To see the nature of the crossover between Halperin's (1,1,1) model and our attractive model, we consider a model Hamiltonian with pseudo potential interaction $V_l^{\uparrow\downarrow} = \cos\theta\delta_{l,0} - \sin\theta\delta_{l,1}$, $V_l^{\uparrow\uparrow} = V_l^{\downarrow\downarrow} = 0$. We calculated the relevant value of θ which has a maximum overlap with the exact ground state as a function of d/l and found a continuum transition from $\theta = 0$ ((1,1,1) model) to $\theta = \pi/2$ (attractive model). This is the decay of correlation hole. When the layer spacing d exceeds $1.4l$, the nature of correlation changes qualitatively. The two-particle correlation functions $g_{\uparrow\uparrow}$ and $g_{\uparrow\downarrow}$ exhibit a $2k_F$-like oscillation. Such a behavior was discussed in a monolayer system with $\nu = 1/2$ by Rezayi and Read [7] where k_F ($= 1/l$ in $\nu = 1/2$ [5]) is a Fermi wave number of the composite fermion.

Finally we consider the evolution of the ground state based on a modified composite fermion theory. This theory of the bilayer system was formulated by Rajaraman. [22] A filling factor $\nu_\mathrm{T} = 1$ system was discussed by several authors. [18,19,23,24] We introduce composite fermion feild; $\phi_\sigma^\dagger(\mathbf{r}) = \psi_\sigma^\dagger(\mathbf{r})\ \mathrm{e}^{J_\sigma(\mathbf{r})}$ where ψ_σ is the electrons field operator and $J_\sigma(\mathbf{r}) = R_{\sigma\sigma'} \int d^2\mathbf{r}'\rho_{\sigma'}(\mathbf{r}')\ln(z - z') - \frac{eB}{4}|\mathbf{r}|^2$ has a role of the flux attachment. [25] The matrix R is appropriately determined by interlayer interaction at a corresponding layer separation. For $\nu_\mathrm{T} = 1$ bilayer system, when the layer separation is sufficiently large, the Fermi-liquid state of the composite fermions with $R_{\sigma\sigma'} = 2\delta_{\sigma,\sigma'}$ corresponds to eq.(2). On the other hand, for small d/l, we should choose $R_{\sigma\sigma'} = 2\delta_{\sigma,-\sigma'}$ because of strong interlayer interaction. These theories are called independent composite fermion and mutual composite fermion theories respectively. The operation of the flux attachment is carried on by a two-body interaction that cause a pairing instability of the composite fermions. [19,23,26] The wave function of the interlayer paired state is written as

$$\Psi = \mathrm{Pf}\ K(\mathbf{r}_i^\sigma - \mathbf{r}_j^{-\sigma})\ \Psi_{R_{\uparrow\uparrow},R_{\downarrow\downarrow},R_{\uparrow\downarrow}}, \tag{9}$$

where $\mathrm{Pf}K = \mathcal{A}(K_{1,2}K_{3,4}\cdots)$ is an antisymmetric product and the factor $K(\mathbf{r}) = \frac{1}{\Omega}\sum_\mathbf{k} \mathrm{e}^{\mathrm{i}\mathbf{k}\cdot\mathbf{r}} \left(\frac{E_\mathbf{k} - \epsilon_\mathbf{k}}{\Delta_\mathbf{k}^*}\right)$ is determined by the procedure of the Bogoliubov transformation. $\Delta_\mathbf{k}$ is a pairing gap, $\epsilon_\mathbf{k} = k^2/2m^* - \epsilon_\mathrm{F}$, and $E_\mathbf{k} = \sqrt{\epsilon_\mathbf{k}^2 + |\Delta_\mathbf{k}|^2}$. Pf $K(\mathbf{r}_i^\sigma - \mathbf{r}_j^{-\sigma})$ is the wave function of the composite fermions that is a real space representation of the BCS-like paired state. [26] When d/l is small, we consider the case $R_{\sigma,\sigma'} = 2\delta_{\sigma,-\sigma'}$. For a week pairing state in the terminology of Read and Green, [27] in the $l_z = -1$ channel $\Delta_\mathbf{k} = \Delta \cdot \left(\frac{k_x - \mathrm{i}k_y}{k_F}\right)$, $K(\mathbf{r}) = 1/z$. Using the Cauchy identity, [28] one obtain $\Psi_{1,1,1}$ from eq.(9). On the other hand, for large d/l case, Kim *et al.* proposed several paired state of the independent composite fermions. [19] However anything of that kind was not found in our calculation. It rather seems that our results of the two-particle correlation function indicate the continuum development of electrons pairing in $l_z = 1$ channel that corresponds to $l_z = -1$ pairing of the mutual composite fermions. This is understood as follows. Note the interlayer Coulomb interaction reduces the interlayer pairing. Starting from $d = 0$, where the mutual composite fermion theory gives $\Psi_{1,1,1}$ as the ground state, in-

crease of d/l makes the interlayer pairing develop further because of the reduction of the interlayer interaction. We note that the increase of d/l also decreases a spin stiffness coefficient and makes the charge gap decrease. [12] In Fig.4, the r-dependence of the function $|K(r)|$ with two case of the intensity of the interlayer pairing $\Delta/\epsilon_{\rm F}$ are plotted. We found that at $\Delta/\epsilon_{\rm F} = \sqrt{2}$, $|K(r)|$ coincides with $1/r$ completely that lead to $\Psi_{1,1,1}$. At $\Delta/\epsilon_{\rm F} = 3$, $|K(r)|$ behaves as $1/r^{1.4}$ asymptoticaly at large r ($> 10l$) and $1/r^2$ at small r ($< 10l$). This $1/r^2$ behavior at small r with eq.(9) is consistent with the contraction of the correlation hole seen in Fig.2(b).

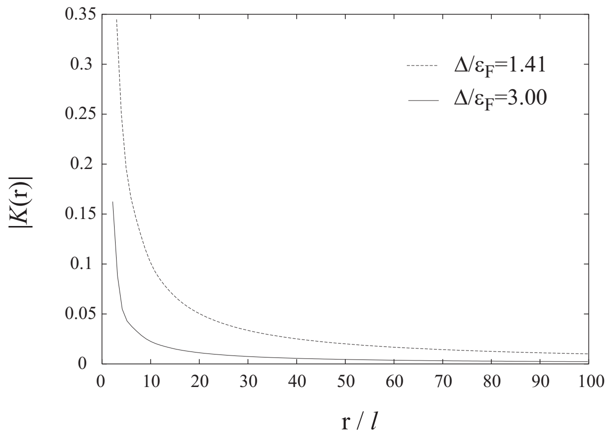

FIG. 4. The r-dependence of the absolute value of $K(\mathbf{r})$ in eq (9).

In this letter we have investigated the evolution of the bilayer quantum Hall ferromagnetic state. Around $d/l \sim 1$ $\Psi_{1,1,1}$ is not relevant as a trial wave function. We proposed an attractive model that has imperfect correlation holes and gives a good approximation to the ground state. In our model state, the electrons are interacting attractively in $l_z = 1$ channel which is connected to the paired state of the mutual composite fermion theory. The crossover corresponds to a decline of the correlation hole that is consistent with the fact that as d/l is increased the peak of the zero-bias conductance and the Hall drag resistance turned down in the experiments. [13,14]

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